THE FIRST WAVE EQUATION MIGRATION RTM WITH DATA CONSISTING OF PRIMARIES AND INTERNAL MULTIPLES: THEORY AND 1D EXAMPLES

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(Received June 28, 2014; revised version accepted July 23, 2014)

ABSTRACT

Liu, F. and Weglein, A.B., 2014. The first wave equation migration RTM with data consisting of primaries and internal multiples: theory and 1D examples. *Journal of Seismic Exploration*, 23: 357-366.

Reverse time migration (RTM) is the cutting-edge imaging method used in seismic exploration. In earlier RTM publications, density was often chosen and used to balance a medium with velocity variation, such that the acoustic impedance - the product of velocity and density - stays constant. Thus, normal incidence reflections from sharp boundaries are avoided. In order to be more complete, consistent, realistic, and predictive, general velocity and density variations (not constrained by impedance matching) are intentionally included in our study so that we can test the impact of reflections on the first wave equation migration RTM algorithms. The major objectives of this article are to advance our understanding and to provide concepts, added imaging capabilities, and new algorithms for RTM. Although our objective of extracting useful subsurface information from recorded data is not different from that of well-known previous RTM publications, our method is different. Although all current methods utilize the wave equation, the imaging condition they call upon, the time and space coincidence of up- and down-going waves, ultimately results in an asymptotic approximate imaging algorithm. All current industry applied RTM algorithms do not correspond to predicting a coincident source and receiver experiment at depth at $t = 0$. That imaging principle is the defining property of wave equation migration (WEM). The method of this paper represents WEM for RTM. In this paper, we present the first WEM RTM imaging tests, with a discontinuous reference medium and outputting the correct image locations and distinct reflection coefficients from above and below each reflector, with primaries and internal multiples in the data. There is "no cross talk" or any other artifacts as reported by other methods that seek to migrate data with primaries and multiples. That is an implementation and analysis of Weglein et al. (2011a,b) with primaries and internal multiples in the data.

KEY WORDS: wave equation migration RTM, Green's theorem, boundary conditions.

0963-0651/14/$5.00

INTRODUCTION

One of the major early objectives of Reverse Time Migration (RTM) is to obtain a better image of salt flanks and subsalt targets through diving waves than can often be obtained by one way migration imaging through the complex overburden. The key new capability of the RTM method compared with one-way migration algorithms is to allow two-way wave propagation in the imaging procedure. This article follows closely the idea established in Weglein et al. (2011a,b): achieving a Green's function with vanishing Dirichlet and Neumann boundary conditions at the deeper boundary, to eliminate the need for measurements at depth.

As stated in, e.g., Whitmore (1983); Baysal et al. (1983); Luo and Schuster (2004); Fletcher et al. (2006); Liu et al. (2009) and Vigh et al. (2009), accurate medium properties above the target are required for RTM and that properly is shared with the new RTM method developed/progressed in this paper. One major difference is that in most RTM algorithms in the industry, a smoothed version of the velocity is used in the imaging procedure to avoid reflections from the velocity model itself, while the exact velocity model is used in the example in this article. We adopt the notations of the aforementioned articles as much as possible while introducing some minor modifications to allow smooth expansion/extension into new territory.

The major contributions of this article are:

- It provides the first example that predicts the source and receiver experiment at depth starting with data that consists of primaries and internal multiples. The new method to predict the experiment in the volume derives from a classic, well-defined and well-understood math-physics starting point. The latter is the essential and defining ingredient for WEM RTM and is realized by developing a Green's function with both Dirichlet and Neumann boundary conditions at the lower surface of the volume.

- It incorporates both velocity and density variation for WEM RTM.

In this paper, G_0^+ and G_0^- are used to denote causal and anti-causal Green's functions, respectively. G_0^{DN} is used to denote the Green's function with vanishing Dirichlet and Neumann boundary conditions at the lower surface of the volume. $k = \omega/c_0$ where c_0 is the constant velocity of the reference medium, and ω is the angular frequency.

THEORY

Green's theorem wave-field prediction with density variation

First, let us assume the wave propagation problem in a (one-dimensional) volume V bounded by a shallower depth A and deeper depth B:

$$\{(\partial/\partial z')[1/\rho(z')](\partial/\partial z') + [\omega^2/\rho(z')c^2(z')]\}D(z',\omega) = 0 \ , \qquad (1)$$

where $A \leq z' \leq B$ is the depth, and $\rho(z')$ and $c(z')$ are the density and velocity fields, respectively., and D is the wave-field. In exploration seismology, we let the shallower depth A be the measurement surface where the seismic acquisition takes place. The volume V is the finite volume defined in the "finite volume model" for migration, the details of which can be found in Weglein et al. (2011a). We measure D at the measurement surface $z' = A$, and the objective is to predict D anywhere between the shallower surface and another surface with greater depth, $z' = B$. This can be achieved via the solution of the wave-propagation equation in the same medium by an idealized impulsive source or Green's function:

$$\{(\partial/\partial z')[1/\rho(z')](\partial/\partial z') + [\omega^2/\rho(z')c^2(z')]\}G_0(z,z',\omega) = \delta(z-z') \ , \qquad (2)$$

where z is the location of the source, and $A < z' < B$ and z increase in a downward direction. Abbreviating $G_0(z,z',w)$ as G_0, the solution for D in the interval $A < z < B$ is given by Green's theorem:

$$D(z,\omega) = [1/\rho(z')]\{D(z',\omega)(\partial G_0/\partial z') - G_0[\partial D(z',\omega)/\partial z']\}\Big|_{z'=A}^{z'=B} \ , \qquad (3)$$

where A and B are the shallower and deeper boundaries, respectively, of the volume to which the Green's theorem is applied. It is identical to equation (43) of Weglein et al. (2011a), except for the additional density contribution to the Green's theorem. Interested readers may find the derivation of equation (3) in section 2 of Liu and Weglein (2013).

Note that in eq. (3), the field values on the closed surface of the volume V are necessary for predicting the field value inside V. The surface of V contains two parts: the shallower portion $z' = A$ and the deeper portion $z' = B$. In seismic exploration, the data at $z' = B$ is not available. For example, one of the significant artifacts of the current RTM procedures is caused by this phenomenon: there are events necessary for accurate wave-field prediction that reach $z' = B$ but never return to $z' = A$. The solution, based on Green's theorem without any approximation, was first published in Weglein et al. (2011a) and Weglein et al. (2011b), the basic idea can be summarized as follows.

Since the wave equation is a second-order differential equation, its general solution has a great deal of freedom/flexibility. In other words, for a wave equation with a specific medium property, there are an infinite number of solutions. This freedom in choosing the Green's function has been taken advantage of in many seismic-imaging procedures. For example, the most popular choice in wave-field prediction is the physical solution G_0^+. In downward continuing a one-way propagating up-going wave field to a point in the subsurface, the anti-causal solution G_0^- is often used in eq. (3).

Weglein et al. (2011a,b) show that (with the G_0^- choice), the contribution from $z' = B$ will be zero under 1 way wave assumptions, and only measurement is required at $z' = A$. For two-way propagating waves, G_0^- will not make the contribution for $z' = B$ vanish. However, if both G_0 and $\partial G_0/\partial z'$ vanish at the deeper boundary $z' = B$, where measurements are not available, then only the data at the shallower surface (i.e., the actual measurement surface) is needed in the calculation. We use G_0^{DN} to denote the Green's function with vanishing Dirichlet and Neumann boundary conditions at the deeper boundary.

Downward continuation of both source and receiver

The original Green's theorem in eq. (3) is derived to downward continue the wave field (i.e., receivers) to the subsurface. It can also be used to downward continue the sources down to the subsurface by taking advantage of reciprocity: the recording is the same after the source and receiver locations are exchanged.

Assuming we have data on the measurement surface: $D(z_g,z_s)$ (its ω dependency is ignored), we can use $G_0^{DN}(z,z_g)$ to downward continue it from the receiver depth z_g to the target depth z:

$$D(z,z_s) = \{[\partial D(z_g,z_s)/\partial z_g]G_0^{DN}(z,z_g) - D(z_g,z_s)\partial G_0^{DN}(z,z_g)/\partial z_g\}/\rho(z_g) \quad (4)$$

Taking the $\partial/\partial z_s$ operation on eq. (4), we have a similar procedure to downward continue $\partial D(z_g,z_s)/\partial z_s$ to the subsurface:

$$\partial D(z,z_s)/\partial z_s = \{[\partial^2 D(z_g,z_s)/\partial z_g \partial z_s]G_0^{DN}(z,z_g) - [\partial D(z_g,z_s)/\partial z_s][\partial G_0^{DN}(z,z_g)/\partial z_g]\}/\rho(z_g) \quad . \quad (5)$$

With eqs. (4) and (5), we downward continue the data D and its partial derivative over z_s to the subsurface location z. According to reciprocity, $D(z,z_s) = E(z_s,z)$, where $E(z_s,z)$ is resulted from exchanging the source and receiver locations in the experiment to generate D at the subsurface. The predicted data

$E(z_s,z)$ can be considered as the recording of receiver at z_s for a source located at z. For this predicted experiment, the source is located at depth z, according to the Green's theorem, we can downward continue the recording at z_s to any depth shallower than or equal to z.

In seismic migration, we downward continue $E(z_s,z)$ to the same subsurface depth z with $G_0^{DN}(z,z_s)$ to have an experiment with coincident source and receiver:

$$\begin{aligned} E(z,z) &= \{[\partial E(z_s,z)/\partial z_s]G_0^{DN}(z,z_s) \\ &\quad - E(z_s,z)[\partial G_0^{DN}(z,z_s)/\partial z_s]\}/\rho(z_s) \quad , \\ &= \{[\partial D(z,z_s)/\partial z_s]G_0^{DN}(z,z_s) \\ &\quad - D(z,z_s)[\partial G_0^{DN}(z,z_s)/\partial z_s]\}/\rho(z_s) \quad . \end{aligned} \tag{6}$$

If $z_s < z_g$ and we assume the data is deghosted, the $\partial/\partial z_s$ operation on $D(z_g,z_s)$ is equivalent to multiplying $-ik$, in this case, eq. (6) can be further simplified:

$$E(z,z) = -[\{[\partial G_0^{DN}(z,z_s)/\partial z_s] + ikG_0^{DN}(z,z_s)\}/\rho(z_s)]/D(z,z_s) \quad . \tag{7}$$

NUMERICAL EXAMPLES

As an example, for a 2-reflector model (with an ideal impulsive source located at z_s, the depth of receiver is $z_g > z_s$, the geological model is listed in Table 1), the data and its various derivatives can be expressed as:

$$\begin{aligned} D(z_g,z_s) &= (\rho_0 x^{-1}/2ik)\{y + \alpha y^{-1}) \quad , \\ \partial D(z_g,z_s)/\partial z_g &= (\rho_0/2)x^{-1}\{y - \alpha y^{-1}) \quad , \\ \partial D(z_g,z_s)/\partial z_s &= -(\rho_0/2)x^{-1}\{y + \alpha y^{-1}) \quad , \\ \partial^2 D(z_g,z_s)/\partial z_g \partial z_s &= (\rho_0 k/2i)x^{-1}\{y - \alpha y^{-1}) \quad , \end{aligned} \tag{8}$$

where $x = e^{ikz_s}$, $y = e^{ikz_g}$, $\sigma = e^{ikz}$, $\alpha = e^{ik(2a_1)}[R_1 + (1 - R_1^2)\beta]$, and $\beta = \Sigma_{n=0}^{\infty}(-1)^n R_1^n R_2^{n+1} e^{ik_1(2n+2)[a_2-a_1]}$.

And $R_1 = (c_1\rho_1 - c_0\rho_0)/(c_1\rho_1 + c_0\rho_0)$, and $R_2 = (c_2\rho_2 - c_1\rho_1)/(c_2\rho_2 + c_1\rho_1)$ are the reflection coefficients from geological boundaries.

Above the first reflector

For $z < a_1$, the boundary values of the Green's function are:

$$\begin{aligned} G_0^{DN}(z,z_g) &= \rho_0[e^{ik(z-z_g)} - e^{ik(z_g-z)}]/2ik = \rho_0(\sigma y^{-1} - \sigma^{-1} y)/2ik \ , \\ G_0^{DN}(z,z_s) &= \rho_0(\sigma x^{-1} - \sigma^{-1} x)/2ik \ , \\ \partial G_0^{DN}(z,z_g)/\partial z_g &= \rho_0(\sigma y^{-1} + \sigma^{-1} y)/-2 \ , \\ \partial G_0^{DN}(z,z_s)/\partial z_s &= \rho_0(\sigma x^{-1} + \sigma^{-1} x)/-2 \ . \end{aligned} \tag{9}$$

After substituting eq. (8) into eq. (7), we have:

$$E(z,z) = \{1 + e^{ik(2a_1 - 2z)}[R_1 + (1 - R_1^2)\beta]\}/(2ik/\rho_0) \ . \tag{10}$$

The result above can be Fourier transformed into the time domain to have:

$$E(z,z,t)/(-\rho_0 c_0/2) = H(t) + R_1 H(t - t_1) + (1 - R_1^2)$$

$$\times \sum_{n=0}^{\infty} (-1)^n R_1^n R_2^{n+1} H[t - t_1 - (2n + 2)t_2] \ , \tag{11}$$

where $t_1 = (2a_1 - 2z)/c_0$ and $t_2 = (a_2 - a_1)/c_1$. Balancing out the $-\rho_0 c_0/2$ factor*, the data after removing the direct wave is denoted as $\hat{D}(z,t) = (-2/\rho_0 c_0)E(z,z,t) - H(t)$:

$$\hat{D}(z,t) = R_1 H(t - t_1) + (1 - R_1^2) \sum_{n=0}^{\infty} (-1)^n R_1^n R_2^{n+1} H[t - t_1 - (2n+2)t_2] \ . \tag{12}$$

Table 1. The properties of an acoustic medium with two reflectors, at depth a_1 and a_2.

Depth Range	Velocity	Density
$(-\infty, a_1)$	c_0	ρ_0
(a_1, a_2)	c_1	ρ_1
(a_2, ∞)	c_2	ρ_2

* This factor is present in the incident wave, i.e., causal Green's function for a homogeneous medium with density ρ_0 and velocity c_0.

We take the imaging condition as first letting $z \to a_1$ through values smaller than a_1, and then (subsequently) taking the limit as $t \to 0^+$, that is, approaching zero from positive values, we find:

$$\lim_{t\to 0^+}\Big(\lim_{z\to a_1^-}\hat{D}(z,t)\Big) = R_1 \quad , \tag{13}$$

where

$$\begin{aligned} a_1^- &= a_1 - \varepsilon_1 \qquad \varepsilon_1 > 0 \quad , \\ 0^+ &= 0 + \varepsilon_2 \qquad \varepsilon_2 > 0 \quad , \end{aligned} \tag{14}$$

and we obtained the image of the first reflector at the actual depth a_1 with the correct reflection coefficient as amplitude.

Between the first and second reflectors

For $a_1 < z < a_2$, we have:

$$\begin{aligned} G_0^{DN}(z,z_g) &= [(R_1\lambda - \lambda^{-1})\mu + (\lambda - R_1\lambda^{-1})\mu^{-1}]/[2ik_1(1 + R_1)/\rho_1] \quad , \\ \partial G_0^{DN}(z,z_g)/\partial z_g &= [(R_1\lambda - \lambda^{-1})\mu - (\lambda - R_1\lambda^{-1})\mu^{-1}]/[\mathbf{2}k_1(\mathbf{1} + \mathbf{R}_1)/\mathbf{k}\rho_1] \quad , \end{aligned} \tag{15}$$

where $\lambda = e^{ik_1(z-a_1)}$, $\mu = e^{\bar{i}k(z_g-a_1)}$, $k_1 = \omega/c_1$. Substituting eq. (15) into eq. (8), and transforming the aforementioned result into the time domain, we have:

$$\begin{aligned} E(z,z,t)/(-\rho_1 c_1/\mathbf{2}) &= H(t) + 2\sum_{n=1}^{\infty}(-1)^n R_1^n R_2^n H\{t - [2n(a_2 - a_1)/c_1]\} \\ &+ \sum_{n=0}^{\infty}(-1)^{n+1} R_1^{n+1} R_2^n H\{t - [2z + 2na_2 - 2(n+1)a_1]/c_1\} \\ &+ \sum_{n=0}^{\infty}(-1)^n R_1^n R_2^{n+1} H\{t - [2(n+1)a_2 - 2na_1 - 2z]/c_1\} \quad . \end{aligned}$$

Balancing out the $-\rho_1 c_1/\mathbf{2}$ factor, the data after removing the direct wave is denoted as $\hat{D}(z,t) = (-2/\rho_1 c_1)E(z,z,t) - H(t)$:

$$\hat{D}(z,t) = 2\sum_{n=1}^{\infty}(-1)^n R_1^n R_2^n H\{t - [2n(a_2 - a_1)/c_1]\}$$

$$+ \sum_{n=0}^{\infty} (-1)^{n+1} R_1^{n+1} R_2^n H\{t - [2z + 2na_2 - 2(n+1)a_1]/c_1\}$$

$$+ \sum_{n=0}^{\infty} (-1)^n R_1^n R_2^{n+1} H\{t - [2(n+1)a_2 - 2na_1 - 2z]/c_1\} \quad ,$$

and after taking the $t = 0^+$ imaging condition, we have:

$$\hat{D}(z,t) = \begin{cases} -R_1 & \text{if } (z = a_1 + \varepsilon_1) \\ 0 & \text{if } (a_1 < z < a_2) \\ R_2 & \text{if } (z = a_2 - \varepsilon_2) \end{cases} , \tag{16}$$

where $\varepsilon_1, \varepsilon_2 \to 0$ and then $t \to 0^+$. Note that in the previous section, i.e., to image above the first reflector at a_1, we obtain the amplitude R_1 when z approaches a_1 from above. In this section we image below the first reflector at a_1, the amplitude of the image is $-R_1$ when z approaches a_1 from below, as it should.

Below the second reflector

For $z > a_1$, the boundary value of the Green's function is:

$$G_0^{DN}(z,z_g) = \{[\nu^{-1}(R_2\lambda - \lambda^{-1}) + R_1\nu(\lambda - R_2\lambda^{-1})]\mu$$

$$+ [R_1\nu^{-1}(R_2\lambda - \lambda^{-1}) + \nu(\lambda - R_2\lambda^{-1})]\mu^{-1}\}$$

$$/[2ik_2(1 + R_1)(1 + R_2)/\rho_2] \quad ,$$

where $\lambda = e^{ik_2(z-a_2)}$, $\mu = e^{ik(z_g-a_1)}$, and $\nu = e^{ik_1(a_2-a_1)}$, $k_2 = \omega/c_2$.

The final downward continuation result can be expressed as:

$$E(z,z) = (\rho_2/2ik_2)[1 - R_2 e^{ik_2(2z-2a_2)} + (1 - R_2^2) e^{ik_2(2z-2a_2)}$$

$$\times \sum_{n=0}^{\infty} (-1)^{n+1} R_1^{n+1} R_2^n e^{ik_1(2n+2)(a_2-a_1)}] \quad .$$

The time domain counterpart of the equation above is:

$$E(z,z,t) = -(\rho_2 c_2/2)\{H(t) - R_2 H[t - (2z-2a_2)/c_2]$$

$$+ (1 - R_2^2) H[t - (2z-2a_2)/c_2 - (2n+2)(a_2-a_1)/c_1] \quad .$$

Balancing out the $-\rho_2 c_2/2$ factor, the data after removing the direct wave is denoted as $\hat{D}(z,t) = (-2/\rho_2 c_2)E(z,z,t) - H(t)$:

$$\hat{D}(z,t) = -R_2 H[t - (2z-2a_2)/c_2]$$
$$+ (1 - R_2^2)H[t - (2z-2a_2)/c_2 - (2n+2)(a_2-a_1)/c_1]$$

and after taking the $t = 0^+$ imaging condition, we have:

$$\hat{D}(z,t) = \begin{cases} -R_2 & \text{if } (z = a_2 + \varepsilon) \\ 0 & \text{if } (a_2 < z) \end{cases}, \tag{17}$$

where $\varepsilon \to 0^+$. Note that in the previous section, i.e., to image between the first and second reflectors, we obtain the amplitude R_2 when z approach a_2 from above. In this section we image below the second reflector at a_2, the amplitude of the image is $-R_2$ when z approaches a_2 from below, as it should.

SUMMARY

Green's theorem provides a solid math-physics foundation to realize that requirement. For two way propagating waves and seismic reflection data, it calls for a Green's function, G_0^{DN}, that vanishes along with its normal derivative, on the lower surface of the volume. Although the expressions of G_0^{DN} in this article are analytic, they have been validated by finite-difference scheme for future generalization of the procedure for a multidimensional earth, the detail can be found in Liu and Weglein (2013).

We also have reported the first wave equation migration RTM imaging tests, with a discontinuous reference medium and images that have the correct depth and amplitude (that is, producing the reflection coefficient at the correctly located target) with primaries and multiples in the data. The current paper is an extension, implementation and analysis of Weglein et al. (2011a,b) with primaries and multiples in the data. There are no artifacts, "cross-talk" or other problems reported in the literature with other methods for migrating primaries and multiples for imaging and/or illumination in Weglein (2014).

To accurately predict structure and reflection coefficient information in cases where two way propagation is necessary calls for wave equation migration (WEM) RTM. WEM requires the production of a source and receiver experiment in the subsurface. The two main circumstances where RTM is called for include: (1) diving waves, in, e.g., presalt plays, and (2) when data with primaries and multiples is being imaged and inverted. The first of these two

circumstances will often use a smooth velocity and density model whereas the latter requires an accurate discontinuous velocity and density model. This paper is the first detailed, transparent and analytic demonstration of how the second of these two applications would be carried out for wave equation migration RTM with primaries and internal multiples in the data. For practical reasons [smooth velocity (achievable) versus discontinuous (unachievable) velocity], the near term added value of this first WEM RTM (compared to all current industry applied asymptotic RTM) will be in the first of these two circumstances, for diving waves, where we anticipate it will provide improved amplitude information at, e.g., the imaged presalt target.

ACKNOWLEDGMENTS

Drs. James D. Mayhan and Jinlong Yang are thanked for assisting in the preparation and editing of this paper. Dr. Mark A. Meier is thanked for useful and constructive comments, that have benefited this paper. M-OSRP sponsors are thanked for their encouragement and their support.

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