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WHY EXACTLY DO MULTIPLES NEED TO BE REMOVED IN DIRECT SEISMIC PROCESSING METHODS? AND WHAT ABOUT INDIRECT METHODS?

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ABSTRACT

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This paper provides a new and detailed analysis on why multiples need to be removed in all current seismic processing methods - without any exception. We cast a wide and inclusive net - covering all direct methods and indirect methods. That includes methods that use multiples to estimate an image of an unrecorded primary, as well as within model-matching methods, for example, FWI. We include methods that require or do not require subsurface information. We conclude that all methods require multiples to be removed, either initially, or eventually within the method, and its application. A new migration method, Stolt-Claerbout III Migration for Heterogeneous and Discontinuous Media, plays an essential and fundamental role in that new insight, understanding and perspective. This is the first paper of a two-paper set, this one explaining "why" multiples are a pressing, and increasingly prioritized necessity and challenge, now and for the foreseeable future. The second paper describes "how" multiples are removed, with a tool-box perspective, and how to make cost-effective choices among options-and a recognition of both recent progress and challenges and open-issues.

KEY WORDS: Multiples, Primaries, imaging, Migration, Direct and Indirect, Continuous and Discontinuous Velocity Migration, FWI and smooth migration velocity, Multiple Removal, Illumination.

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INTRODUCTION

Multiple removal has been a long-term objective in seismic exploration. Recent methods that use multiples for different processing goals and objectives can be worthwhile. However, their use can also be a source of confusion as to whether the removal is no longer essential, a priority, or even necessary since some may now view multiples as "rehabilitated" and sit along- side primaries as entirely useful events. One purpose of this paper is to disabuse us of that seriously flawed and erroneous thinking and to understand that the use of multiples and the removal of multiples have the same exact goal and objective: the imaging of primaries. We explain exactly why multiples need to be removed in all direct and indirect seismic processing methods - and therefore why multiple removal remains a key and central issue and challenge in seismic exploration.

Multiple Removal and Direct and Indirect Seismic Processing Methods

In what follows, we will explain why all direct and indirect seismic processing methods remove multiples at some step or stage within the method. There is often a blur and avoidance among seismic processing researchers about the conceptual and practical differences between direct and indirect methods. Among references that define and distinguish between direct and indirect seismic processing methods are: Weglein (2013, 2017, 2018).
We can view this paper as the first of a two-part set, with the first one here (basically asking "why") and the second paper asking "how?"

DIRECT SEISMIC METHODS

In this paper, we will start with direct methods, and then separate direct methods by whether they require or do not require a velocity model. Then we will move on to an analysis for indirect methods- and finally, the overall conclusion for both all direct and indirect methods.

Direct seismic methods with a velocity model – migration in homogeneous, continuous and discontinuous velocity models

Migration and migration-inversion are the two main industry processing methods for locating and analyzing structure, and they are direct and require a velocity model and subsurface information, respectively.

Methods that employ the wave equation for migration have two ingredients: (1) a wave propagation concept and (2) an imaging condition. In his landmark paper, Claerbout (1971), described three imaging conditions for seismic migration. He combined these imaging conditions with one-way wave propagation concepts to determine structure at depth. The three imaging conditions are: (1) the exploding reflector model, (2) the space and time coincidence of an upgoing wave from the reflector and a down going wave from the source, and (3) the predicted coincident source and receiver experiment at depth, at time equals zero. We will refer to these original imaging conditions as Claerbout I, II and III. Claerbout I (the exploding reflector model) only relates to stacked or zero offset data. Claerbout II and Claerbout III are valid for pre-stack data. The third imaging condition, CIII, stood alone in terms of clarity, physical interpretation and definitiveness and in its potential to be extended for complex structure and associated amplitude analysis.

For example, Stolt and Weglein (1985) and Stolt and Weglein (2012) extended the original CIII for more physically complete and accommodating structural models, and in addition provide an automatic detailed angle dependent amplitude analysis at the target, for both specular and non-specular reflections. We label the latter extension ofClaerbout III or CIII to image and perform subsequent amplitude analysis, that is, migration-inversion, for both simple and complex curved and pinch-out structure as Stolt-Claerbout III migration or SCIII. To arrange for inversion after migration, or migration-inversion, Stolt and Weglein, 1985, relaxed the coincident source receiver condition, within the original Claerbout III imaging principle, to retain the lateral offset dependence in the predicted experiment at depth, at t= 0, resulting in the Stolt-Claerbout III imaging principle. That migration-inversion extension of Claerbout III assumed one-way propagation in the prediction of the experiment at depth.

Recently, CIII and SCIII was further extended (Weglein et al, 2016) for two -way wave propagation, in the prediction of the experiment at depth, to accommodate both a smoothly varying medium without one way wave assumptions, and discontinuous media. That recent advance allowed, for the first time, imaging above and beneath a reflector in a discontinuous medium, the latter required to analyze and to unambiguously define the roles of primaries and multiples in migration, with recorded data from a discontinuous medium, (when using SCIII with an accurate discontinuous velocity or a smooth approximate velocity model. The predicted coincident source and receiver experiment at depth consists of all the events that experiment would record, if you actually had a source and receiver at that subsurface location.

Weglein et al. (2016) provided that extension of SCIII to accommodate a discontinuous medium and for the first time, to image above and below each reflector, without any artifacts or issues such as “rabbit ears”. That new SCIII Migration for a Discontinuous Medium (or for a Continuous Medium Without Making One Way Wave Assumptions), is represented in eq. (3) below.

Stolt Claerbout III migration in homogeneous, or smoothly varying media (with a one-way wave propagation assumption).

For one-way wave propagation in a homogeneous or smoothly varying 2D medium, the latter with an assumed one-way wave propagation, the predicted source and receiver experiment at depth,
$D\ (x_g, z_g, x_s, z_s, \omega)$ is

$$D\left(x_g, z_g, x_s, z_s, \omega\right)(\text{at depth}) = \int_{S_s} \frac{\partial G_0^{-D}(x_s, z_s, x_s', z_s', \omega)}{\partial z_s'} \times \int_{S_g} \frac{\partial G_0^{-D}\left(x_g, z_g, x_g', z_g', \omega\right)}{\partial z_g'} \times D\left(x_g', z_g', x_s', z_s', \omega\right) dS_g\, dS_s, \qquad (1)$$

where $(x_g,\ z_g)$ and $(x_s,\ z_s)$ are the coordinates of the predicted receiver and source at depth, and D in the integrand is the data, D (on the measurement surface), G_0^{-D} is the anticausal Green's function with a Dirichlet boundary condition on the measurement surface, where *s* connotes shot, and *g*, receiver, respectively.

The primed variables are measurement surface coordinates. The extension of eq. (1) [and eqs. (2) and (3) below] to a three-dimensional medium is straightforward, (with data $D\left(x_g', y_g', z_g', x_s', y_s', z_s', \omega\right)$. When assuming one way wave propagation, and choosing the anticausal Green's function, the lower surface doesn't contribute to the field predicted inside the finite volume, from surface measurements.

The new Stolt Claerbout III migration to accommodate a discontinuous medium without artifacts (i.e., with no "rabbit ears") and for a smoothly varying medium (<u>without</u> a one-way wave propagation assumption)

For two-way propagation, in, e.g., a discontinuous medium above the image point (that is, above the target reflector), we begin with the recorded data $D\ (x_g^l,\ z_g^l,\ x_s^l,\ z_s^l,\ \omega)$ on horizontal measurement surfaces, with z_g^l = constant and z_s^l = constant. The first part of the predicted experiment, uses Green's theorem, and sums over measurement surface receivers- and outputs the receiver at x_g, z_g, at depth, while keeping the source at x_s^l, z_s^l (on the measurement surface) in eq. (2) below.

$$\int \{ \frac{\partial G_0^{DN}}{\partial z_g'} (x_g, z_g, x_g', z_g', \omega)\, D(x_g', z_g', x_s', z_s', \omega)$$
$$- G_0^{DN}(x_g, z_g, x_g', z_g', \omega)\, \frac{\partial D}{\partial z_g'} (x_g', z_g', x_s', z_s', \omega)\} \underbrace{\{dx_g'\}}_{\{dS_g\}} =$$
$$D(x_g, z_g, x_s', z_s', \omega) \qquad (2)$$

A second application of Green's theorem inputs (2) and then predicts the experiment for both the receiver at x_g, z_g and the source at x_s, z_s, at depth using eq. (3) below

$$\int \{ D(x_g, z_g, x_s', z_s', \omega)\, \frac{\partial G_0^{DN}}{\partial z_s'} (x_s, z_s, x_s', z_s', \omega)$$
$$- G_0^{DN}(x_s, z_s, x_s', z_s', \omega)\, \frac{\partial D}{\partial z_s'} (x_g, z_g, x_s', z_s', \omega)\} \underbrace{\{dx_s'\}}_{\{dS_s\}}$$
$$= D(x_g, z_g, x_s, z_s, \omega) \qquad (3).$$

Eq. (3) is the prediction of the experiment at depth required for Stolt-Claerbout III Migration in a discontinuous medium. Please see Weglein et al, 2016 for the detail on the extensions (in concept and capability) within equation (3) to accommodate any geometry in structure (including curved surfaces and pinch-outs) and to then automatically perform amplitude analysis. Those forms of Equation (3) provide the current high-water mark of migration and inversion (migration-inversion) capability and effectiveness. That suite of capabilities is not possible to achieve with Claerbout II imaging, and its RTM and least-squared RTM extensions.

G_0^{DN} in Equation (3) is the Green's function for discontinuous media (in a finite volume) that (in order to not need data at the lower surface at some finite depth in the earth) is arranged to vanish along with its normal derivative on the lower surface of the finite volume. Please note that $dS_g = dx'_g$ and $dS_s = dx'_s$ in a 2D prediction, with data on a line. An integral of eq. (3) with $z_g = z_s$ over ω produces the predicted experiment at t = 0 and SCIII migration.

Summary for Stolt CIII migration for Heterogeneous and Discontinuous Media in a layered medium with data consisting of primaries and multiples

The analytic analysis of eq. (3) for a layered medium is found in Weglein (2016), [following Liu and Weglein (2014)] and demonstrates, for the first time, how the actual individual recorded events (within the recorded data on the measurement surface) contribute to the predicted coincident source and receiver experiment at depth, and to each individual event in that predicted experiment. The predicated experiment at depth consists of its own primaries and multiples, with its own predicated singly or multiply reflected events, respectively. That analysis in Weglein (2016) output the source and receiver experiments predicted both above and below each reflector.

At each depth z, below the measurement surface, the predicted coincident source and receiver experiment cares about (depends on) *all* the actual recorded primary and multiple events on the measurement surface. When examining the predicted coincident source and receiver experiment, at any point at depth, that doesn't correspond to a location above or beneath a reflector, the above cited analytic analysis (Weglein, 2016) produces a zero result when the $t = 0^+$ imaging condition is applied. However, when using an accurate discontinuous velocity model and the SCIII heterogeneous media migration eq. (3) with the imaging condition and $t = 0^+$ is applied to the coincident source and receiver experiment at depth, z, above or beneath a reflector then only the recorded primaries on the measurement surface contribute to the migration result – and the SCIII imaging with a coincident source receiver experiment at depth, at t=0, results in a predicted primary that depends only on a recorded primary on the measurement surface, within a coincident source-receiver prediction at depth, at t=0. The multiples in the recorded data, contribute to the predicted coincident experiment at depth, below and above each reflector. But the contributions from the recorded multiples arrive at positive times, in the predicted coincident experiment and have no contribution at t=0. Only a primary in the recorded data contributes to the primary in the predicted experiment, and gives a non-zero result for the SCIII migration with a discontinuous accurate velocity.

The conclusion: multiples do not contribute to the image at any depth, when using SCIII eq. (3) with an accurate discontinuous velocity model above, and beneath, the reflector to be imaged. That is, if we migrated data consisting of primaries and multiples with an accurate discontinuous velocity model, and used Stolt CIII migration for heterogeneous media, eq. (3) at t = 0, then multiples in the recorded data on the measurement surface will not contribute to the image above or below a reflector. For that (correct discontinuous migration velocity) situation, multiples would not cause false images, and would not be harmful, or helpful. Please see Figs. 1 - 3 that illustrate these steps and conclusions for recorded data consisting of primaries and free surface multiples, and Figs. 4 - 7 for recorded data consisting of primaries and internal multiples, using eq. (3) a Stolt Claerbout III migration for heterogeneous media (Weglein et al., 2016).

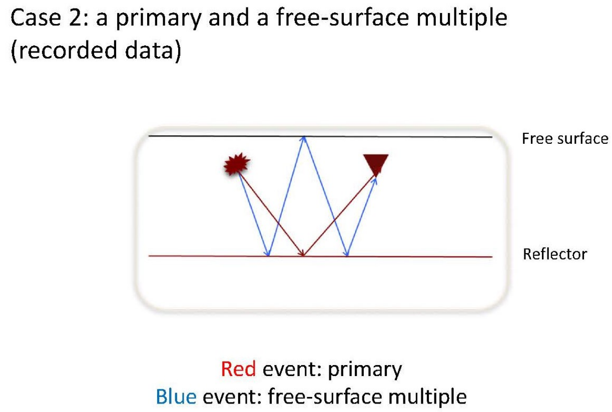

Fig. 1. A primary and a free surface multiple (recorded data).

However, if the migration (in a discontinuous medium) uses a smooth velocity for the data consisting of primaries and multiples, the "predicted" source and receiver experiment at depth will not be the actual source and receiver experiment at depth. That difference and error results in every multiple causing a false image.

Hence, for a smooth velocity model, multiples must be removed. Since the industry leading edge migration velocity methods can, at best, find an improved smooth velocity model - and there are currently no candidates to produce an accurate discontinuous velocity model, recorded multiples must be removed now and for the foreseeable future. See, e.g., the 2021 SEG/DGS Workshop on Velocity Model Building (Saad et al., 2021) and the final/wrap-up presentation by Weglein. [Weglein, 2021]

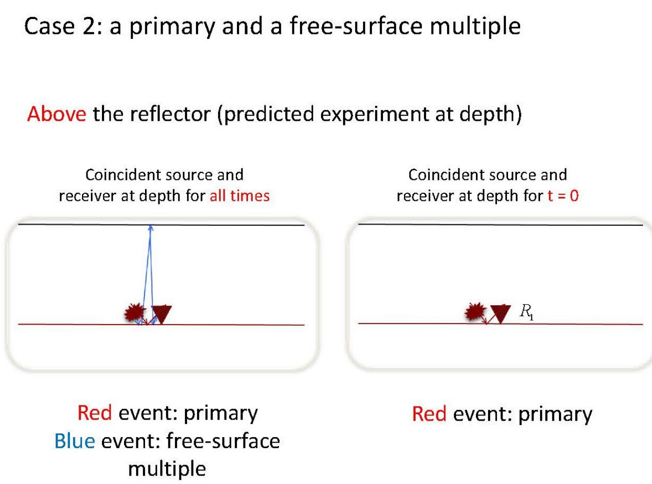

Fig. 2. The predicted experiment (and the t = 0 image) at depth (above the reflector) from a recorded data consisting of a primary and a free-surface multiple.

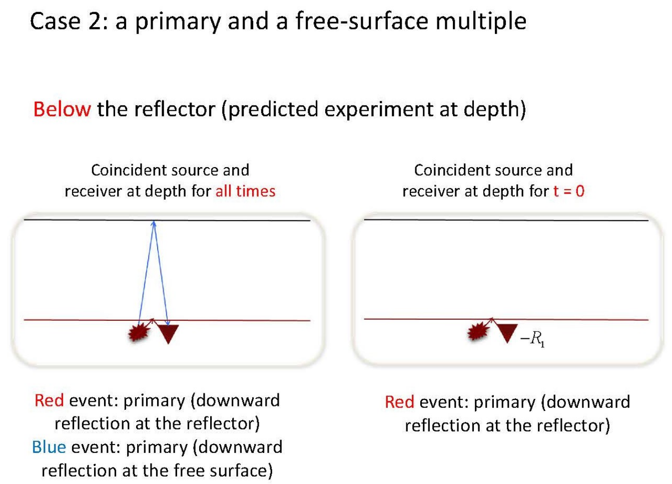

Fig. 3. The predicted experiment (and t = 0 image) at depth beneath the reflector, from a recorded data consisting of a primary and free surface multiple.

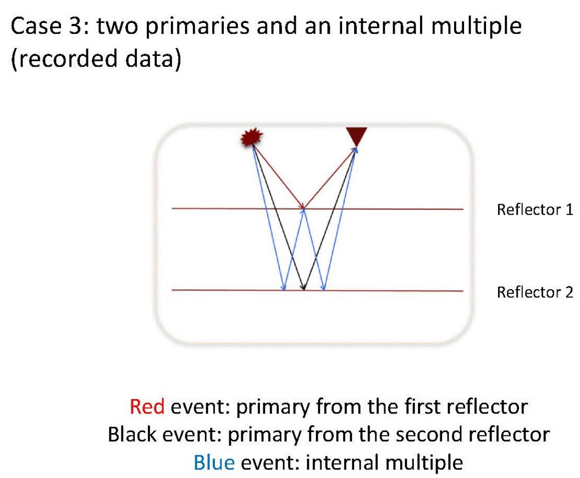

Fig. 4. The recorded data consisting of two primaries and an internal multiple.

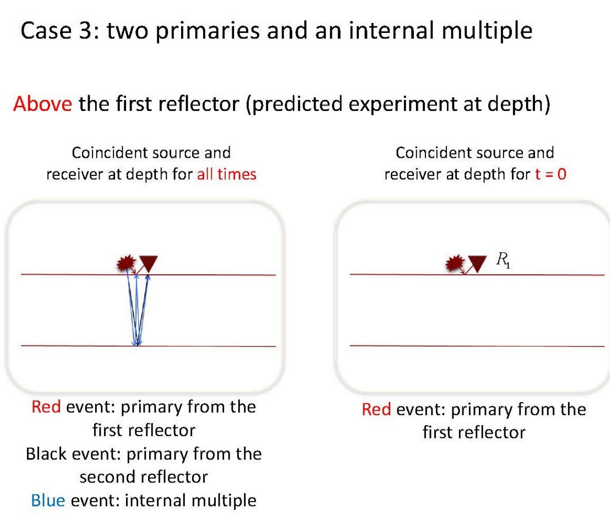

Fig. 5. The predicted experiment (and image) above the first reflector, for a recorded data consisting of two primaries and an internal multiple.

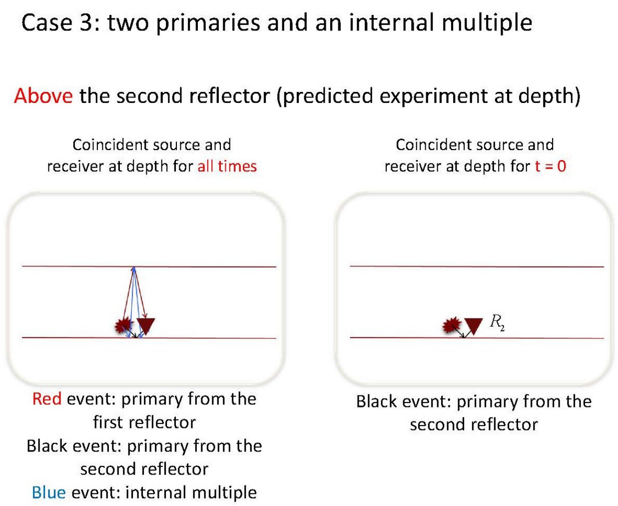

Fig. 6. The predicted experiment (and image at t = 0) above the second reflector for a recorded data consisting of two primaries and an internal multiple.

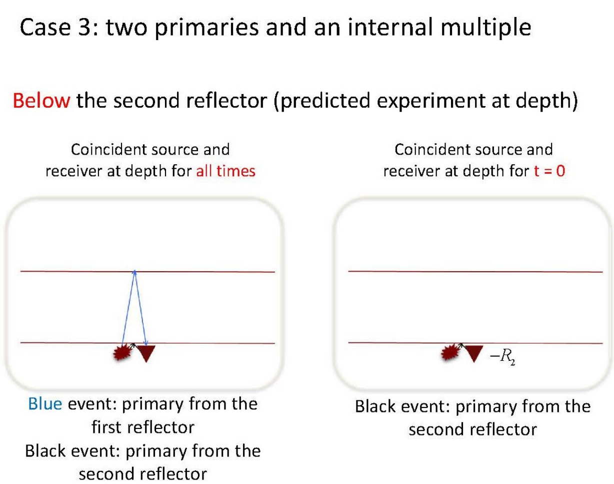

Fig. 7. The predicted experiment at depth (and image at t = 0) beneath the second reflector for a recorded data consisting of two primaries and an internal multiple.

Direct methods with a velocity model: removing and using multiples

In the previous section we reviewed a new (and first) migration method (Weglein et al., 2016; Zou et al., 2017; Weglein, 2016) that can image above and below reflectors in a discontinuous medium, (without artifacts) and can accommodate primaries and multiples. The conclusion of that analysis is when using that new migration (SCIII migration for heterogeneous and discontinuous media) with an accurate discontinuous velocity, multiples provide no harm or benefit, and there is no reason to remove them. However, when using a smooth velocity model, multiples will cause false images that can interfere with or masquerade as reflectors. The latter analysis is not possible with any of the other current methods of migration, e.g., RTM, since RTM cannot image in a discontinuous medium (without artifacts) even with an accurate discontinuous velocity model. The methods that seek to remove these intrusive RTM artifacts (see e.g., Liu et al., 2011) have their own serious artifacts that damage the structural and amplitude fidelity of images. SCIII for heterogeneous media, eq. (3), images recorded data consisting of primaries and multiples, in a discontinuous medium without artifacts. And for all migration methods that employ a smooth velocity model, multiples will cause problems, always producing false images, and often interfering with structure and damaging amplitude analysis.

Using multiples to estimate the image of unrecorded primaries

All of the migration methods we have been discussing assume that the recorded data coverage is adequate to carry out their function. What about when the set of recorded primaries is inadequate?

Multiples can at times be useful, see e.g., Lu et al. (2011), Whitmore et al. (2011a, b), Ma and Zou (2015) to aid the missing unrecorded primaries issue. However, it is extremely important to recognize that that use of multiples never corresponds to migrating a multiple. I n fact, the migration of a multiple has no meaning, absolutely no meaning whatsoever (Weglein, 2019b).

Within the sphere of direct seismic methods with a velocity model, certain multiples can be useful to seek an approximate image of an unrecorded primary that is an unrecorded subevent of the recorded multiple. Constrained by our ability to find (at most) a smooth velocity model for migration, the removal of recorded multiples is necessary to image recorded primaries, and the removal of unrecorded multiples is required to find an approximate image of an unrecorded primary. The assumption is that the unrecorded subevent of a recorded multiple is an unrecorded primary. That, in turn, assumes that the unrecorded subevent is not an unrecorded multiple. Hence, unrecorded subevents that are unrecorded multiples must be removed. All recorded and unrecorded multiples must be removed to image primaries, recorded and unrecorded primaries, respectively. Weglein (2017, 2018, 2019a).

The above methodology (of using multiples) assumes, e.g., that a recorded free surface multiple consists of two subevents, one that is recorded, and that the second subevent is a primary that is unrecorded. The idea is to extract and predict, from the recorded multiple and its recorded subevent, the approximate image of the unrecorded primary. If all the subevents of the multiple are recorded, the multiple has no use. This use of multiples is itself a testament to the fact that a complete set of recorded primaries is sufficient for imaging the subsurface

What about Illumination?

We often hear that multiples can be useful to enhance illumination. To paraphrase Jon Claerbout "Waves, and the reflected seismic wavefield, are ubiquitous, and have no illumination issues. However, seismic processing methods that are asymptotic high frequency approximations, ray-like in nature, (e.g., Kirchhoff and RTM migration, see Weglein et al., 2016) can "squeeze" the wave into ray paths, that leave gaps and produce illumination issues and challenges." In contrast, Stolt CIII migration (Weglein et al., 2016), eq. (1) and eq. (3), are the only migration methods that make no high frequency approximation in either the imaging condition or the propagation model.

In summary: The fact that our most capable migration velocity models (today and for the foreseeable future) are smooth and continuous, remains the key and central reason that all multiples must be removed for imaging and inversion when using any method that requires a velocity model.

Direct methods without a velocity model

The only direct method that can input primaries and multiples, and output all processing objectives without knowing, estimating or determining sub- surface information (including velocity) is the isolated task subseries of the inverse scattering series. There are distinct subseries that directly remove free surface and internal multiples (e.g., Weglein et al., 2003; Zou et al., 2019).
If multiples were needed to perform tasks such as depth imaging, Q compensation and parameter estimation the inverse scattering series would not have subseries whose entire purpose is to remove them. For the only direct seismic inverse method that does not require a velocity model, (the isolated task specific inverse scattering subseries algorithms) multiples must be removed, with primaries (only) as input to the imaging and inversion subseries. See, e.g., Weglein et al. (2012), Zhang and Weglein (2009a, b), Liang et al. (2013), Zou and Weglein (2018).

Indirect seismic methods, e.g., CIG flatness, AVO and FWI

There are different types of indirect inverse methods. Among them are:
(1) seeking to satisfy a property that an inverse solution would possess;
(2) solving a forward problem in an inverse sense, and
(3) model matching.

The CIG flatness criteria are in the first category, while solving an elastic inverse in terms of PP data, e.g., AVO, and FWI are in the second and third category, respectively. Why each of these is "indirect" is fully detailed in Weglein (2013, 2018, 2017, 2020).

Indirect methods like CIG flatness represent a necessary but not sufficient imaging condition that a correct migration velocity model would satisfy. The CIG flatness criteria assumes that the data consists of primaries and that multiples have been removed.

References for CIG are Anderson et al. (2012), Baumstein et al. (2009), Ben-Hadj-Ali et al. (2008, 2009), Biondi and Sava (1999), Biondi and Symes (2004), Brandsberg-Dahl et al. (1999), Chavent and Jacewitz (2011), Fitchner (2011), Guasch et al. (2012), Kapoor et al. (2012), Rickett and Sava (2002), Sava et al. (2005), Sava and Fomel (2003), Sirgue et al. (2009, 2010, 2012), Symes and Carazzone (1991), Tarantola (1987), Zhang and Biondi (2013). Many wrong velocity models can and will also satisfy a flat common-image-gather criterion, especially under complex imaging circumstances.

Another type of indirect method, FWI, is a model matching methodology that can input any data set, consisting of primaries, free surface multiples and internal multiples. Among FWI references are Brossier et al. (2009), Crase et al. (1990), Gauthier et al. (1986), Nolan and Symes (1997), Pratt (1999), Pratt and Shipp (1999), Sirgue et al. (2010), Symes (2008), Tarantola (1984, 1986), Valenciano et al. (2006), Vigh and Starr (2008), Zhou et al. (2012). In practice, primaries are considered not enough, not full enough, and primaries and all multiples are apparently too much to match, a bit too full. And matching primaries and free surface multiples, are the perfect degree of fullness. Therefore, within current FWI practice, internal multiples are first removed and then primaries and free surface multiples are matched. Hence, an internal multiple removal is called for in FWI.

THE OUTPUT FROM FWI IS (AT BEST) A SMOOTH VELOCITY MODEL, AND ALL MULTIPLES NEED TO BE REMOVED WHEN MIGRATING WITH A SMOOTH VELOCITY MODEL

As was documented in a recent SEG/DGS Workshop on Velocity Model Building Saad et al. (2021) and the final/wrap-up presentation by Weglein (2021), FWI has been useful in providing an improved smooth velocity for migration. As we pointed out earlier in this paper, with a smooth migration velocity model, all multiples must be removed. Hence, within FWI today internal multiples need to be removed, and the use of the smooth velocity output from FYI, require all multiples to be removed in the use of that velocity in migration methods.

Regarding AVO: AVO is a first term in a modeling equation for PP data run backwards- and, hence, is not a direct method, and assumes that multiples have been removed before the Zoeppritz equations are applied to estimate the relative in changes in earth mechanical properties.

Therefore, either initially or ultimately all multiples must be removed in all indirect seismic methods.

We suggest the videos in the link below
http://youtube.com/playlist?list=PL4ITzyY3tPVenlpnBQRJKurK5bvmw6XWs
to complement the above analysis and conclusions.

CONCLUSIONS

All current migration velocity analysis methods can (at best) produce a smooth continuous migration velocity model. For direct seismic methods, that require subsurface information, for example, migration with a smooth velocity model, all multiples will cause false images that can masquerade as or interfere with structure - and need to be removed. To clearly analyze the role of primaries and multiples in imaging requires a new form of migration (that we label Stolt Claerbout III for heterogeneous media) that can image in a discontinuous medium without artifacts. With an accurate discontinuous velocity model, the new Stolt-Claerbout III Migration for heterogeneous media, we showed that multiples cause no harm and provide no benefit. If (in the future) we could find an accurate discontinuous velocity model and used the Stolt-Claerbout III Migration for discontinuous media, we would have no reason to remove multiples. However, currently and for the foreseeable future, we are confined to (at best) improving a smooth approximate velocity, (e.g., output from FWI) and hence the absolute need to remove all multiples, before migration for structure and amplitude analysis, remains in place and of very high priority.

To use a recorded multiple to estimate the RTM image of an unrecorded primary, we assume the recorded multiple consists of two subevents, one recorded and the other not recorded. Let's further assume that the unrecorded subevent is an unrecorded primary. Then the recorded multiple, and the recorded subevent of the multiple, are used to estimate the image of an unrecorded primary subevent of the multiple. To satisfy the latter assumption, unrecorded subevents of the recorded multiple, that are (not unrecorded primaries but rather) unrecorded multiples, must be removed- since the unrecorded event is migrated with a form of RTM using a smooth velocity model. Furthermore, the original recorded multiple must be removed to image recorded primaries, again with a smooth velocity model. Hence, recorded and unrecorded multiples must be removed to image recorded and unrecorded primaries, respectively.

The inverse scattering series is the only direct inversion method for a multi-dimensional earth, and in addition it doesn't require any subsurface information (including velocity) to be known, estimated or determined. It contains distinct isolated task subseries that remove free surface and internal multiples. Only primaries are called for in task specific subseries for structure determination, parameter estimation and Q compensation, the latter without knowing, estimating or determining Q. If multiples were needed in the only direct multidimensional inverse method, the inverse scattering series, to achieve imaging and parameter estimation and Q compensation, it would not contain isolated task subseries whose sole purpose and existence is designed to remove them. Direct methods are purposeful, and do not remove events that are needed to carry out its purposes.

For indirect methods, based on satisfying a criterium that only relate to primaries, e.g., CIG flatness, multiples must first be removed.

FWI is model matching of primaries and multiples and currently is able (at best) to output a smooth velocity model for migration. Multiples must be removed when using a smooth velocity for migration. For the smooth migration velocity output of FWI to be useful, for imaging and inversion, multiples must first be removed.

Hence, all direct and indirect seismic processing methods require all multiples to be removed, either initially or eventually.

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